

Transmission system for transmitting a multilevel signal

The invention relates to a transmission system for transmitting a multilevel signal from a transmitter to a receiver.

The invention further relates to a transmitter for transmitting a multilevel signal, a receiver for receiving a multilevel signal, a mapper for mapping an interleaved encoded signal according to a signal constellation onto a multilevel signal, a demapper for demapping a multilevel signal according to a signal constellation, a method of transmitting a multilevel signal from a transmitter to a receiver and to a multilevel signal.

In transmission systems employing so-called bit interleaved coded modulation (BICM) schemes a sequence of coded bits is interleaved prior to being encoding to channel symbols. Thereafter, these channels symbols are transmitted. A schematic diagram of a transmitter 10 which may be used in such a transmission system is shown in Fig. 1. In this transmitter 10 a signal comprising a sequence of information bits $\{b_k\}$ is encoded in a Forward Error Control (FEC) encoder 12. Next, the encoded signal $\{c_k\}$ (i.e. the output of the encoder 12) is supplied to an interleaver 14 which interleaves the encoded signal by permuting the order of the incoming bits $\{c_k\}$. The output signal $\{i_k\}$ of the interleaver 14 (i.e. the interleaved encoded signal) is then forwarded to a mapper 16 which groups the incoming bits into blocks of m bits and maps them to a symbol set consisting of 2^m signal constellation points with corresponding labels. The resulting sequence of symbols $\{x_k\}$ is a multilevel signal which is transmitted by the transmitter 10 over a memoryless fading channel to a receiver 20 as shown in Fig. 2. In Fig. 1 the memoryless fading channel is modeled by the concatenation of a multiplier 17 and an adder 19. The memoryless fading channel is characterized by a sequence of gains $\{\gamma_k\}$ which are applied to the transmitted multilevel signal by means of the multiplier 17. Furthermore, the samples of the transmitted multilevel signal are corrupted by a sequence $\{n_k\}$ of Additive White Gaussian Noise (AWGN) components which are added to the multilevel signal by means of the adder 19. This generic channel model fits, in particular, multicarrier transmission over a frequency selective channel, where the set of instances $k = 1, \dots, N$ corresponds to N subcarriers. Therefore, it falls under the scope of the existing standards for broadband wireless (such as ETSI BRAN HIPERLAN/2, IEEE 802.11a and their advanced versions currently being in

standardization). The main distinguishing feature of BICM schemes is the interleaver 14 which spreads the adjacent encoded bits c_k over different symbols x_k , thereby providing the diversity of fading gains γ_k within a limited interval of the sequence $\{c_k\}$ of coded bits. This yields a substantial improvement in the FEC performance in fading environments. (Pseudo-
5)random interleavers may be used that for big block size N guarantee a uniform spreading and therefore a uniform diversity over the whole coded sequence. Alternatively, row-column interleavers may be used.

It is now assumed that the receiver 20 has a perfect knowledge of the fading gains $\{\gamma_k\}$. This assumption is valid as in practice these gains can be determined very
10 accurately (e.g. by means of pilot signals and/or training sequences). The standard decoding of a BICM-encoded signal has a mirror structure to the structure of the transmitter 10 as shown in Fig. 1. For each k , the received samples y_k and the fading gains γ_k are used to compute the so-called a posteriori probabilities (APP) of all 2^m signal constellation points for x_k . These APP values are then demapped, i.e. transformed to reliability values of individual
15 bits of the k -th block. The reliability value of a bit may be computed as a log-ratio of the APP of this bit being 0 over the APP of this bit being 1, given the set of APP values of 2^m constellation points for the k -th block. Sometimes the APP of a bit being 0 or 1 is replaced by the bitwise maximum likelihood (ML) metrics, i.e. the largest APP over the constellation points matching this bit value. In this way the numerical burden can be reduced. These
20 reliability values are deinterleaved and forwarded to a FEC decoder which estimates the sequence of information bits, e.g. by means of standard Viterbi decoding.

The main drawback of this standard decoding procedure, as compared to the (theoretically possible but impractical) optimal decoding comes from the fact that there is no simultaneous use of the codeword structure (imposed by FEC) and the mapping structure.

25 Although the strictly optimal decoding is not feasible, the above observation gives rise to a better decoding procedure that is illustrated in the receiver 20 as shown in Fig. 2. The basic idea of this procedure is to iteratively exchange the reliability information between the demapper 22 and the FEC decoder 32. The iterative procedure starts with the standard demapping as described above. The reliability values $\{L_k^{(\mu)}\}$ of the demapped bits, after
30 deinterleaving by a deinterleaver 26, serve as the inputs to a soft-input soft-output (SISO) decoder 32 which produces the (output) reliabilities $\{L_k^{(o)}\}$ of the coded bits $\{c_k\}$ that take into account the (input) reliabilities of the demapped bits and the FEC structure. The standard SISO decoders are maximum a posteriori (MAP) decoders, a simplified version of which is

known as a max-log-MAP (MLM) decoder. The difference between the inputs and the outputs of the SISO decoder 32 (often referred to as extrinsic information) is determined by a subtracter 30 and reflects the reliability increment which is the result of the code structure.

This differential reliability is interleaved by an interleaver 28 and used as an a priori

5 reliability during the next demapping iteration. In a similar way, the differential reliability is computed at the successive demapper output (by means of subtracter 24). This reliability represents a refinement due to the reuse of the mapping and signal constellation structure; it is used as an a priori reliability for the subsequent SISO decoding iteration. After the last iteration, the SISO output reliabilities $\{ L_k^{(b)} \}$ of the information bits are fed to a slicer 34 to

10 produce final decisions $\{ \hat{b}_k \}$ on the information bits.

An important feature of the BICM scheme is the mapping of bits according to a signal constellation comprising a number of signal points with corresponding labels. The most commonly used signal constellations are PSK (BPSK, QPSK, up to 8-PSK) and 4-QAM, 16-QAM, 64-QAM and sometimes 256-QAM. Furthermore, the performance of the system depends substantially on the mapping design, that is, the association between the signal points of the signal constellation and their *m-bit* labels. The standard Gray mapping is optimal when the standard (non-iterative) decoding procedure is used. Gray mapping implies that the labels corresponding to the neighboring constellation points differ in the smallest possible number of *m* positions, ideally in only one. An example of a 16-QAM signal constellation with the Gray mapping (*m* = 4) is shown in Fig. 3A. It can easily be seen that the labels of all neighboring signal points differ in exactly one position.

However, the use of alternative mapping designs or mappings may improve dramatically on the performance of BICM schemes whenever any version of the iterative decoding is exploited at the receiver. In European patent application number 0 948 140 an iterative decoding scheme as shown in Fig. 2 is used with what is referred to as anti-Gray encoding mapping. It is however not clear what is meant by this anti-Gray encoding mapping. In a paper entitled "Trellis-coded modulation with bit interleaving and iterative decoding" by X. Li and J. Ritcey, IEEE Journal on Selected Areas in Communications, volume 17, pages 715 to 724, April 1999, a noticeable performance improvement is achieved 25 by means of a widely used mapping design known as the Set Partitioning (SP) mapping. An example of a 16-QAM signal constellation with the SP mapping is shown in Fig. 3B.

In European patent application number 0 998 045 and European patent application number 0 998 087 an information-theoretic approach to mapping optimization is disclosed. The core idea of this approach is to use a mapping that reaches the optimal value of the mutual information between the label bits and the received signal, averaged over the 5 label bits. The optimal mutual information depends on the signal-to-noise ratio (SNR), the design number of iterations of the decoding procedure as well as on the channel model. The optimal value of the mutual information is the value that minimizes the resulting error rate. According to this approach, selection of the optimal mappings relies upon simulations of error rate performance versus the aforementioned mutual information for a given SNR, 10 number of iterations and channel model, with the subsequent computation of mutual information for all candidate mappings. Such a design procedure is numerically intensive. Moreover, it does not guarantee optimal error rate performance of the system. Besides the standard Gray mapping, in these European patent applications two new mappings for 16-QAM signal constellations are proposed (which mappings will be referred to as optimal 15 mutual information (OMI) mappings). 16-QAM signal constellations with these OMI mappings are shown in Figs. 3C and 3D.

It is an object of the invention to provide an improved transmission system for transmitting a multilevel signal from a transmitter to a receiver. This object is achieved in the 20 transmission system according to the invention, said transmission system being arranged for transmitting a multilevel signal from a transmitter to a receiver, wherein the transmitter comprises a mapper for mapping an input signal according to a signal constellation onto the multilevel signal, and wherein the receiver comprises a demapper for demapping the received 25 multilevel signal according to the signal constellation, wherein the signal constellation comprises a number of signal points with corresponding labels, and wherein $D_a > D_f$, with D_a being the minimum of the Euclidean distances between all pairs of signal points whose corresponding labels differ in a single position, and with D_f being the minimum of the Euclidean 30 distances between all pairs of signal points. The Euclidean distance between two signal points is the actual ('physical') distance in the signal space between these two signal points. By using a signal constellation with a D_a which is larger than D_f a substantially lower error rate can be reached than by using any of the prior art signal constellations. Ideally, D_a is as large as possible (i.e. D_a has a substantially maximum value), in which case the error rate

is as low as possible. D_a is referred to as the effective free distance of the signal constellation and D_f is referred to as the exact free distance of the signal constellation.

It is observed that iterative decoding procedures approach the behavior of an optimal decoder when the SNR exceeds a certain threshold. This means that at a relatively high SNR (that ensures a good performance of the iterative decoding) one may assume that an optimal decoder is performing the decoding.

Consider an optimal decoder. In practice, trellis codes are used as FEC for noisy fading channels such as (concatenated) convolutional codes. A typical error pattern is characterized by a small number of erroneous coded bits $\{c_k\}$ at error rates of potential interest. The number of erroneous coded bits is typically a small multiple of the free distance of the code; this number is only a small fraction of the total number of coded bits. The free distance of a code is the minimum number of bits (bit positions) in which two different codewords of the code can differ. Due to interleaving, these erroneous coded bits are likely to be assigned to different labels and therefore different symbols. More specifically, the probability of having only one erroneous coded bit per symbol approaches one along with the increase of the data block size.

Hence, the overall error rate (for error rates of potential interest) is improved when the error probability is decreased for such errors that at most one bit per symbol is corrupted. This situation can be reached by maximizing the minimum D_a of the Euclidean distances between all pairs of signal points whose corresponding labels differ in a single bit position.

In an embodiment of the transmission system according to the invention $\overline{H_1}$ has a substantially minimum value, with $\overline{H_1}$ being the average Hamming distance between all pairs of symbols corresponding to neighboring signal points. The Hamming distance between two labels is equal to the number of bits (bit positions) in which the labels differ. By this measure, an accurate decoding of the multilevel signal in the receiver is reached at a relatively small SNR. A typical feature of iterative decoding is a relatively poor performance up to some SNR threshold. After this threshold, the error rate of the iterative decoding approaches the performance of an optimal decoder quite soon, along with the increase of the SNR. It is therefore desirable to decrease this SNR threshold value. This threshold value depends on the starting point of the iterative procedure, i.e. on the distribution of the reliability values $L_k^{(\mu)}$ provided by the demapper on the first iteration. The worst reliability values are due to the neighboring signal points, therefore the 'average' number of coded bits

that suffer from these poor reliabilities is proportional to the 'average' number of positions in which the labels, which correspond to the neighboring signal points, are different. In other words, the SNR threshold degrades (i.e. increases) along with the increase of the average Hamming distance between the labels that are assigned to the neighboring signal points.

- 5 Ideally, $\overline{H_1}$ is as small as possible, i.e. $\overline{H_1}$ has a minimum value, for which value of $\overline{H_1}$ the SNR threshold will also be minimal.

The above object and features of the present invention will be more apparent from the following description of the preferred embodiments with reference to the drawings, wherein:

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Fig. 1 shows a block diagram of a transmitter according to the invention,

Fig. 2 shows a block diagram of a receiver according to the invention,

Figs. 3A to 3D show prior-art 16-QAM signal constellations,

- 15 Fig. 4 shows graphs illustrating the packet error rate versus E_b/N_0 (i.e. the SNR per information bit) for several 16-QAM mappings,

- 20 Fig. 5 shows graphs illustrating the bit error rate versus E_b/N_0 for several 16-QAM mappings,

- Fig. 6 shows graphs illustrating the packet error rate versus E_b/N_0 for a standard 8-PSK signal constellation and for a modified 8-PSK signal constellation,

- 25 Fig. 7 shows graphs illustrating the bit error rate versus E_b/N_0 for a standard 8-PSK signal constellation and for a modified 8-PSK signal constellation,

Figs. 8A to 8G show improved 16-QAM signal constellations,

Figs. 9A to 9C and Fig. 10 show improved 64-QAM signal constellations,

25 Figs. 11A and 11B show improved 256-QAM signal constellations,

Figs. 12A to 12C show improved 8-PSK signal constellations,

Fig. 13 shows a modified 8-PSK signal constellation.

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In the Figs, identical parts are provided with the same reference numbers.

In Figs 8A to 8G, 9A to 9C, 10, 11A and 11B no horizontal I-axis and vertical Q-axis are shown. However, in these Figs. a horizontal I-axis and a vertical Q-axis must be considered to be present, which I-axis and Q-axis cross each other in the center of each Fig. (similar to the situation as shown in Figs. 3A to 3D).

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The transmission system according to the invention comprises a transmitter 10 as shown in Fig. 1 and a receiver 20 as shown in Fig. 2. The transmission system may comprise further transmitters 10 and receivers 20. The transmitter 10 comprises a mapper 10 for mapping an input signal i_k according to a certain signal constellation onto a multilevel signal x_k . A multilevel signal comprises a number of groups of m bits which are mapped onto a real or complex signal space (e.g. the real axis or the complex plane) according to a signal constellation. The transmitter 10 transmits the multilevel signal x_k to the receiver 20 over a memoryless fading channel. The receiver 20 comprises a demapper 22 for demapping the received multilevel signal (y_k) according to the signal constellation. The signal constellation comprises a number of signal points with corresponding labels. The (de)mapper is arranged for (de)mapping the labels to the signal constellation points such that $D_a > D_f$, with D_a being the minimum of the Euclidean distances between all pairs of signal points whose corresponding labels differ in a single position (these labels may be referred to as Hamming neighbors), and with D_f being the minimum of the Euclidean distances between all pairs of signal points. Such a mapping is referred to as a far neighbor (FAN) mapping.

Now the error rate performance of an iteratively decoded BICM scheme using the prior-art signal constellations as shown in Figs 3A to 3D will be compared with the error rate performance of an iteratively decoded BICM scheme using the 16-QAM FAN signal constellation as shown in Fig. 8E. The FEC coder 12 makes use of the standard 8-state rate (1/2) recursive systematic convolutional code with the feed-forward and feedback polynomials 15₈ and 13₈, respectively. A sequence of 1000 information bits produces, after encoding, random interleaving and mapping, a set of $N = 501$ symbols of 16-QAM that are transmitted over a Rayleigh channel with mutually independent gains $\{\gamma_k\}$. Note that this scenario fits a broadband multicarrier BICM scheme with a very selective multipath channel.

At the receiver 20, an iterative decoding procedure is applied according to the scheme as shown in Fig. 2. In this example we use simplified (ML) reliability metrics for the demapping, along with a standard MLM SISO decoder. A pseudo-random uniform interleaver has been used. The simulation results are shown in Figs. 4 and 5. Fig. 4 shows the packet error rate (PER) versus E_b/N_o and Fig. 5 shows the bit error rate versus E_b/N_o . As expected, the signal constellation of Fig. 3A with the Gray mapping gives the worst results at desirably low error rates (see graphs 48 and 58). The state-of-the art signal constellation with SP mapping as shown in Fig. 3B improves substantially on this result (see graphs 44 and 54). The signal constellation with OMI mapping according to Fig. 3C (see graphs 46 and 56) has a poor packet error rate as compared to the signal constellation with SP mapping. However,

the signal constellation with OMI mapping of Fig. 3D (see graphs 42 and 52) improves substantially on the SP mapping. The signal constellation with FAN mapping as shown in Fig. 8E (see graphs 40 and 50) provides a 2 dB gain at low error rates (specifically at PER $\leq 10^{-3}$) over the best of the prior-art signal constellations.

5 The effective free distance D_a is the minimum of the Euclidean distances taken over all pairs of signal points whose labels differ in one position only. Note that D_a is lower bounded by the exact free distance D_f which is the minimum Euclidean distance over all pairs of signal points. \overline{H}_1 is defined as the average Hamming distance between the pairs of labels assigned to neighboring signal points (i.e. the signal points that are separated from each other
10 by the minimum Euclidean distance D_f). Now \overline{H}_l is defined as the average Hamming distance between all pairs of labels assigned to the l -th smallest Euclidean distance. By the l -th smallest Euclidean distance, the l -th element of an increasing sequence is meant, which sequence consists of all Euclidean distances between the signal points of a given
15 constellation. Note that such a definition of \overline{H}_l is consistent with the definition of \overline{H}_1 . In some cases, joint optimization of the first criterion (i.e. having a D_a which is as big as possible but at least larger than D_f) and the second criterion (i.e. having a substantially minimum \overline{H}_l) yields a set of solutions and some of them have different \overline{H}_l for some $l > 1$. In such cases, the set of solutions may be reduced in the following way. For each l increasing from 1 to m , only the solutions that provide the minimum of \overline{H}_l are retained. This
20 approach reduces the SNR threshold of the iterative decoding process.

All possible signal constellations may be grouped into classes of equivalent signal constellations. The signal constellations from the same equivalence class are characterized by the same sets of Euclidean and Hamming distances. Therefore all signal constellations of a given equivalence class are equally good for our purposes.

25 There are some obvious ways to produce an equivalent signal constellation to any given signal constellation. Moreover, the total number of equivalent signal constellations that may be so easily inferred from any given signal constellation is very big. The equivalence class of a given signal constellation is defined as a set of signal constellations that is obtained by means of an arbitrary combination of the following operations:

- 30 (a) choose an arbitrary binary m -tuple and add it (modulo 2) to all labels of the given signal constellation;
(b) choose an arbitrary permutation of the positions of m bits and apply this permutation to all the labels;

(c) for any QAM constellation, rotate all signal points together with their labels by $l\frac{\pi}{2}$, $1 \leq l \leq 3$;

(d) for any QAM constellation, swap all signal points together with their labels upside down, or left to the right, or around the diagonals;

- 5 (e) for PSK, rotate all signal points together with their labels by an arbitrary angle.

A smart algorithm has been designed to accomplish the exhaustive classification of all possible signal constellations for 16-QAM for which D_a has a maximum value. This exhaustive search resulted in seven signal constellations which are shown in Figs.

- 10 8A to 8G. It is easy to show that all these signal constellations achieve the maximum possible effective free distance D_a which equals $\sqrt{5}D_f$. Note that all the prior-art signal constellations only achieve $D_a = D_f$.

In terms of the second criterion (i.e. having a substantially minimum \overline{H}_1), the signal constellations of Figs. 8A to 8G have the respective \overline{H}_1 values

- 15 $\left\{2\frac{1}{6}, 2\frac{1}{3}, 2\frac{1}{3}, 2\frac{1}{3}, 2\frac{1}{6}, 3, 2\frac{1}{3}\right\}$. Note that the signal constellations of Figs. 8A and 8E yield the

minimum of \overline{H}_1 . Moreover, it can be shown that for 16-QAM, $\overline{H}_1 = 2\frac{1}{6}$ is the minimum possible \overline{H}_1 that may be achieved whenever $D_a > D_f$. Therefore, the signal constellations of Figs. 8A and 8E (and the signal constellations belonging to the equivalence classes thereof) jointly optimize both criteria under the condition $D_a > D_f$.

- 20 Since the total number of signal constellations grows very fast along with increasing m (For example, the total number of signal constellations is $2.1 \cdot 10^{13}, 2.6 \cdot 10^{35}$ and $1.3 \cdot 10^{99}$, respectively, for $m = 4, 5$ and 6 , respectively) the exhaustive search for the best signal constellation is not feasible for $m > 4$. In such cases, an analytic construction should be found that allows to either simplify the exhaustive search or to restrict
- 25 ourselves to a limited set of signal constellations that contain 'rather good' ones.

As a matter of fact, $2^m - QAM$ is almost the only signaling which is practically used for $m \geq 4$. For this signaling, with even m , we specify a family of linear signal constellations as follows:

Let $m=2r$, then the $2^m - QAM$ signaling represents a regular two-dimensional grid with 2^r points in the vertical and horizontal dimensions. A set of labels $\{L_{i,j}\}_{i \in I, j \in J}$ is defined wherein $L_{i,j}$ is a binary m -tuple that stands for the label of the signal point with the vertical coordinate i and the horizontal coordinate j . A signal constellation will be called
 5 linear if and only if

$$L_{1,1} = O_m, \quad L_{i,j} = L_{i,1} \oplus L_{1,j}, \quad 1 \leq i, j \leq 2^r \quad (1)$$

wherein O_m is the all-zero m -tuple and \oplus denotes the modulo 2 addition.

This family of signal constellations is of interest because of the observation
 that all the signal constellations in the Figs. 8A-8G, except for the signal constellations in the
 10 Figs. 8B and 8C, appear to be linear. These linear signal constellations as well as the
 following sub-family of linear signal constellations may also be constructed without applying
 the above mentioned (first and second) design criteria.

A sub-family of linear signal constellations can be obtained via the following
 equation:

$$L_{i,1} = X_i A, \quad L_{j,1} = Y_j A, \quad 1 \leq i, j \leq 2^r \quad (2)$$

where $\{X_i\}_{i \in I}$ and $\{Y_j\}_{j \in J}$ are two arbitrary sets of binary m -tuples and A is an arbitrary
 $m \times m$ matrix with binary inputs which is an invertible linear mapping in the m -dimensional
 linear space defined over the binary field with the modulo 2 addition.

The use of (2) allows to confine the exhaustive search over all possible linear
 20 signal constellations to a search over the sets $\{X_i\}, \{Y_j\}$. For a given pair of sets $\{X_i\}, \{Y_j\}$ and
 the desired D_a , a suitable A can easily be determined.

An exhaustive search within the sub-family (2) for 64-QAM led to the
 following results: 12 equivalence classes were found with $D_a = \sqrt{20}D_f$, which is the upper
 bound on D_a for 64-QAM. The further minimization of \overline{H}_1 reduced this set to 3 equivalence
 25 classes. All these classes achieve $\overline{H}_1 = 2 \frac{3}{14}$. The corresponding signal constellations are
 shown in Figs. 9A to 9C.

Within the sub-family (2) signal constellations were searched that minimize
 \overline{H}_1 under the condition $D_a > D_f$. For 64-QAM the theoretical minimum of \overline{H}_1 is defined
 by the lower bound $\overline{H}_1 \geq 2 \frac{1}{14}$. No signal constellations with $\overline{H}_1 < 2 \frac{3}{14}$ were found for

$D_a > \sqrt{17}D_f$. For $D_a = \sqrt{17}D_f$ there are 57 equivalence classes with $\bar{H}_1 = 2\frac{1}{14}$. Among

those, a unique equivalence class was found that minimizes \bar{H}_2 . This class achieves

$$\bar{H}_2 = 2\frac{13}{49}; \text{ it is shown in Fig. 10.}$$

The following material on linear signal constellations is related to various

5 signal constellations for $r > 3$. For those cases, it was not possible to classify all possible signal constellations nor to establish the upper bound on D_a . For 256-QAM, a limited search within the sub-family (2) of linear signal constellations led us to a set of 16 equivalence classes that achieve $D_a = \sqrt{80}D_f$ and $\bar{H}_1 = 2\frac{1}{10}$. Among these 16 classes, we retained only

10 two classes that minimize \bar{H}_2 , achieving thereby $\bar{H}_2 = 2\frac{59}{75}$. Their respective signal constellations are given in Figs 11A and 11B.

For the general case of 2^{2r} -QAM, a sub-set of (2) was designed with which the effective free distance

$$D_a \geq \sqrt{5}2^{r-2}D_f \quad (3)$$

can be reached. This particular construction is described now. First of all, we restrict ourselves to the sets of $\{X_i\}_{1 \leq i \leq 2^r}$ and $\{Y_j\}_{1 \leq j \leq 2^r}$ such that:

(a) the first r bits of X_i represent $(i-1)$ in a binary notation whereas the following r bits are zeros.

(b) the first r bits of Y_j are zeros whereas the following r bits represent $(j-1)$ in a binary notation.

20 For sake of simplicity, this selection of $\{X_i\}$ and $\{Y_j\}$ will be referred to as lexico-graphical. For 64-QAM ($m = 6, r = 3$), the lexico-graphical sets are as follows:

$$\{X_1, X_2, \dots, X_8\} = \{000000, 001000, 010000, 011000, 100000, 101000, 110000, 111000\}$$

$$\{Y_1, Y_2, \dots, Y_8\} = \{000000, 000001, 000010, 000011, 000100, 000101, 000110, 000111\}$$

The advantage of the lexico-graphical selection is twofold. First, it ensures that 25 $(X_i + Y_j) \neq 0_m$ for all $1 \leq i, j \leq 2^r$ except for $i=j=1$, thereby ensuring that all $L_{i,j}$ are different. Second, it allows to easily find A that satisfies (3). To do that, we need to ensure that for every pair $(L_{i,j}, L_{i',j'})$ such that $(L_{i,j} \oplus L_{i',j'})$ has only one non-zero bit, the corresponding signal points are at least D_a apart. Note that the total number of binary m -

tuples having only one non-zero bit is m . These labels are represented by the rows of the $m \times m$ identity matrix I_m such that $(I_m)_{ii} = 1$ for all i and the other elements of I_m are zeros. Due to the linearity conditions (1) and (2), we have $(L_{i,j} \oplus L_{i',j'}) = ((X_i \oplus X_{i'}) \oplus (Y_j \oplus Y_{j'}))A$ for all $1 \leq i, j \leq 2^r$. Once again due to linearity, the matrix A can

5 be uniquely defined by the equation

$$_m = ZA, \text{ where } Z = \{Z_i^T, Z_{i'}^T\}^T \quad (4)$$

is a $m \times m$ matrix with binary inputs which is an invertible linear mapping in the m -dimensional linear space defined over the binary field with the modulo 2 addition (here T denotes matrix transpose). We need to select m linearly independent m -tuples (row vectors)

10 Z_i such that (3) is satisfied.

Of interest are all possible signal constellations that satisfy (1), (2) and (4) with $\{Z_i\}_{1 \leq i \leq m}$ chosen so as to meet (3). According to (4), A will be given by the inverse of Z which, along with the lexicographical selection of $\{X_i, Y_j\}$, (1) and (2), specifies a signal constellation for the desired equivalence class.

15 Let us specify one particular selection of $\{Z_i\}_{1 \leq i \leq m}$ and show that (3) holds: choose the set of $\{Z_i\}$ as an arbitrary ordering of all possible m -tuples that have 2 or 3 non-zero entries of which 2 (mandatory) non-zero entries are always at the first and the $(r+1)$ -st position. Check that there are exactly m -tuples and that all they are linearly independent so that Z is invertible. We now show that (3) holds. Assume that $L_{i,j}, L_{i',j'}$ are two arbitrary labels

20 that differ in one position only. Consequently,

$$L_{i,j} \oplus L_{i',j'} = (I_m)_l \quad (5)$$

i.e. the l -th row of I_m , for some l from $\{1, 2, \dots, m\}$. According to (1) and (2), we may write

$$L_{i,j} \oplus L_{i',j'} = ((X_i \oplus X_{i'}) \oplus (Y_j \oplus Y_{j'}))A \quad (6)$$

Taking into account (4), (5), (6) and the fact that A is invertible, we find

$$(X_i \oplus X_{i'}) \oplus (Y_j \oplus Y_{j'}) = Z_l \quad (7)$$

Recall that, according to the lexicographical ordering, all X_i, Y_j have zeros within the first (last) r positions. Let us inspect the spacings between the pairs (i,j) (i',j') of signal points that satisfy (7) with the aforementioned selection for $\{Z_i\}$. First, consider the single possible m -tuple with only 2 mandatory non-zero entries. Check that (7) yields

$$30 (X_i \oplus X_{i'}) = \overbrace{100\dots0}^{(r-1)\text{times}} \overbrace{00\dots0}^{(r)\text{times}}, \quad (Y_j \oplus Y_{j'}) = \overbrace{00\dots0}^{(r)\text{times}} \overbrace{0100\dots0}^{(r-1)\text{times}}$$

$$(X_i \oplus X_{i'})$$

Note that the first r -bits of $(X_i \oplus X_{i'})$ and the last r -bits of $(Y_i \oplus Y_{i'})$ read 2^{r-1} in binary notation. According to the lexico-graphical selection of $\{X_i\}$ and $\{Y_j\}$, the corresponding signal points (i,j) (i',j') have vertical and horizontal offsets of 2^{r-1} positions. The resulting Euclidean distance between these points is composed of vertical and horizontal distances of

$$5 \quad 2^{r-1} D_f.$$

Now, consider all m -tuples with 3 non-zero entries such that the third (non-mandatory) entry is one of the first r entries. We have

$$(X_i \oplus X_{i'}) = 1 \overbrace{??\dots?}^{\text{(r-1)times}} \overbrace{00\dots0}^{\text{(r)times}}, \quad (Y_j \oplus Y_{j'}) = \overbrace{00\dots0}^{\text{(r)times}} \overbrace{100\dots0}^{\text{(r-1)times}},$$

where $1??\dots?$ has one non-zero entry within the last $(r - 1)$ entries. Using again the properties of the lexico-graphical selection, one can show that this yields a vertical offset of at least

$$10 \quad \frac{1}{2} 2^{r-1} = 2^{r-2} \text{ between the signal points } (i,j) \text{ } (i',j') \text{ whereas the horizontal offset remains } 2^{r-1}.$$

Clearly, the role of vertical/horizontal offsets exchange when we consider such Z_i that the third (non-mandatory) entry is one of the last r entries.

We see that, in all situations, the Euclidean distance between the signal points, whose labels differ in one position only, is composed of vertical and horizontal distances so that one of them equals $2^{r-1} D_f$ and the other one is not less than $2^{r-2} D_f$. Consequently, the minimum of the total Euclidean distance between such points satisfies

$$15 \quad D_a \geq \sqrt{(2^{r-1} D_f)^2 + (2^{r-2} D_f)^2} = 2^{r-2} D_f \sqrt{2^2 + 1} = \sqrt{5} 2^{r-2} D_f \quad (8)$$

The non-linear family of signal constellation classes described hereafter may be seen as an extension of the linear family (1). This family comes from the equivalence classes (b) and (c) of all possible optimal classes for 16-QAM, (see Figs. 8A to 8G) that do not fall within the linear family. We noticed that the equivalence classes (b) and (c) may be regarded as being part of the family defined below.

Let S be a set of binary m -tuples that is closed under the (modulo 2) addition. We define an extension of the family of Fig. 8 as a collection of all equivalence classes of signal constellations having a set of labels $\{L_{i,j}\}_{1 \leq i,j \leq 2^r}$ satisfying

$$25 \quad L_{i,j} = 0_m, \quad L_{i,j} = L_{i,1} \oplus L_{1,j} \oplus f(L_{i,1} \oplus L_{1,j}), \quad 1 \leq i, j \leq 2^r \quad (9)$$

where f is a mapping from the set of m -tuples into itself such that firstly, for any m -tuple x from S , $f(x)$ is also in S and secondly, $f(x) = f(y)$ for any m -tuples x, y such that $(x \oplus y)$ is in S .

For 8-PSK, an exhaustive search was used to find the set of appropriate signal constellations. Apparently, there exist only three equivalence classes that satisfy $D_a > D_f$.

These classes achieve $D_a = 1.84776D_f$ and one of those has $\overline{H}_1 = 2\frac{1}{2}$ while the remaining two achieve $\overline{H}_1 = 2\frac{1}{4}$. The corresponding signal constellations are shown in Figs 12A to 12C.

The success of the new strategy is based on the fact that coded bits are interleaved in such a way that the erroneous bits stemming from (typical) error events end up in different labels with a high probability. This property is ensured statistically when a random interleaver is used with a very big block size N . However, the probability of having more than one erroneous bit per label/symbol is different from zero when N is finite.

This observation leads to the following undesirable effect: the error floor (*i.e.*, the error rate flattening region) will be limited by a non-negligible fraction of error events that are characterized by more than one erroneous bit per label. In such cases, the potential gain due to high D_a will not be realized.

There exists a simple way to overcome the impact of such undesirable error events: the interleaver 14 should ensure that, for every label, the smallest number of the trellis sections (of the underlying FEC) between all pairs of the channel bits contributing to this label, is not less than a certain $\delta > 0$.

Such a design criterion ensures that a single error event may result in multiple erroneous bits per label if and only if this error event spans at least δ trellis sections. For big δ , the corresponding number of erroneous bits of this error event approximately equals $(\delta/2R)$, where R is the FEC rate. By choosing δ big enough, we increase the Hamming distance of such undesirable error events thereby making them virtually improbable. Thus, choosing δ big allows us to control the error floor irrespectively to the block size N . In our simulations a uniform random interleaver was used that satisfies this design criterion with $\delta \geq 25$.

The following result is based on our earlier observation that the effective free distance D_a of a BICM scheme may be substantially bigger than the exact free distance D_f ,

provided that FEC with interleaving and an appropriate signal constellation (i.e. having a D_a which is larger than D_f) is used. Hence, it makes sense to design signal constellations that aim at increasing D_a rather than D_f .

This is supported by the following example. A new signal constellation is

- 5 derived from the standard 8-PSK signal constellation. Let us consider an instance of the new strategy represented by the signal constellation as depicted in Fig. 12C. A standard 8-PSK signal constellation is characterized by $D_a^{8-PSK} = (1 - \cos(\pi/4))^{-1/2} D_f^{8-PSK} \approx 1.84776 D_f^{8-PSK}$. It can easily be seen that this minimum distance is defined by the distances between the signal points within the pairs labeled (000,001), (110,111), (100,101) and (010,011). Indeed, these
10 are the only pairs such that the signal points are separated by the (minimum) rotation of $(\pi/2)$ and their labels differ in one position. Note that D_a may be increased, e.g., by simply rotating (note that rotation preserves a highly desirable constant envelope property of PSK) the signal points labeled {001,111,101,011} (i.e. the second label within each pair) leftwise with a rotation angle θ . An improved signal constellation with $\theta = (3\pi/32)$ is shown in Fig.
15 13, wherein the empty bullets stand for the original places of the rotated points. This signal constellation achieves

$$D_a = \sqrt{((1 + \sin(3\pi/32))/(1 - \cos(\pi/4)))} D_f^{8-PSK} = \sqrt{(1 + \sin(3\pi/32))} D_a^{8-PSK} \approx 1.135907 D_a^{8-PSK}$$

In Figs. 6 and 7 the performance of the modified 8-PSK signal constellation of Fig. 13 (see graphs 60 and 70) is compared with the standard 8-PSK signal constellation of Fig. 12C (see graphs 62 and 72). Note that the modified 8-PSK signal constellation leads to a slight degradation of at most 0.2dB at low SNR. This degradation is compensated by a gain of around 1dB at higher SNR. The modified 8-PSK signal constellation shows better performance at packet error rates below 10^{-2} .

The scope of the invention is not limited to the embodiments explicitly
25 disclosed. The invention is embodied in each new characteristic and each combination of characteristics. Any reference signs do not limit the scope of the claims. The word "comprising" does not exclude the presence of other elements or steps than those listed in a claim. Use of the word "a" or "an" preceding an element does not exclude the presence of a plurality of such elements.